

# 1. Stegreifaufgabe aus der Mathematik

Klasse 11

## - Lösungen -

$$1. \quad a) \quad \lim_{x \rightarrow \infty} \frac{x}{1+x^2} = \lim_{x \rightarrow \infty} \left( \frac{1}{\frac{1}{x} + x} \right) = \underline{\underline{0}} \quad \Bigg| \quad \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$b) \quad \lim_{x \rightarrow \infty} \frac{2x-1}{x} = \lim_{x \rightarrow \infty} \left( 2 - \frac{1}{x} \right) = \underline{\underline{2}}$$

$$c) \quad \lim_{x \rightarrow \pm\infty} \frac{2+x}{x^2} = \lim_{x \rightarrow \pm\infty} \left( \frac{2}{x^2} + \frac{1}{x} \right) = \underline{\underline{0}}$$

$$d) \quad \lim_{x \rightarrow \pm\infty} \frac{x^5}{x^4+1} = \lim_{x \rightarrow \pm\infty} \left( \frac{x}{1+\frac{1}{x^4}} \right) = \underline{\underline{\pm\infty}}$$

$$e) \quad \lim_{x \rightarrow 1} \frac{x^2-1}{x-1} = \lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x \rightarrow 1} (x+1) = \underline{\underline{2}}$$

$$f) \quad \lim_{x \rightarrow -2} \frac{5(x^2-1)}{x+1} = 5 \cdot \lim_{x \rightarrow -2} \frac{(x+1)(x-1)}{x+1} = 5 \cdot \lim_{x \rightarrow -2} (x-1) = 5 \cdot (-3) = \underline{\underline{-15}}$$

$$g) \quad \lim_{x \rightarrow 0} \frac{x^2+4x-8}{\cos x} = \frac{\lim_{x \rightarrow 0} (x^2+4x-8)}{\lim_{x \rightarrow 0} \cos x} = \frac{-8}{1} = \underline{\underline{-8}}$$

$$h) \quad \lim_{x \rightarrow \infty} \frac{4x+1}{2x+1} = \lim_{x \rightarrow \infty} \frac{2(2x+1)-1}{2x+1} = \lim_{x \rightarrow \infty} \left( 2 - \frac{1}{2x+1} \right) = 2 - 0 = \underline{\underline{2}}$$

$$i) \quad \lim_{x \rightarrow \infty} \frac{3x^2-1}{x^2-4} = \lim_{x \rightarrow \infty} \frac{3(x^2-4)+11}{x^2-4} = \lim_{x \rightarrow \infty} \left( 3 + \frac{11}{x^2-4} \right) = 3 + 0 = \underline{\underline{3}}$$

## - Lösungen -

2. geg.:  $f(x) = \frac{2x-1}{5x^2+1} \sqrt{3+\sin 2x}$

$$g(x) = \frac{2x-1}{5x^2+1} \cdot 2 = \frac{4x-2}{5x^2+1}$$

$$\lim(g(x)) \frac{4x-2}{5x^2+1} = \frac{\frac{4}{x} - \frac{2}{x^2}}{5 + \frac{1}{x^2}} = 0$$

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Eine weitere Lösungsvariante wird hier gezeigt:

geg.:  $f(x) = \frac{2x-1}{5x^2+1} \sqrt{3+\sin 2x}$

$$\sqrt{2} \leq \sqrt{3 + \underbrace{\sin 2x}_{-1 \leq y \leq 1}} \leq 2$$

$$g(x) \leq f(x) \leq h(x)$$

bzw.

$$\lim_{x \rightarrow \infty} g(x) \leq \lim_{x \rightarrow \infty} f(x) \leq \lim_{x \rightarrow \infty} h(x)$$

$$\lim_{x \rightarrow \infty} \frac{2x-1}{5x^2+1} \cdot \sqrt{2} \leq \lim_{x \rightarrow \infty} f(x) \leq \lim_{x \rightarrow \infty} \frac{2x-1}{5x^2+1} \cdot 2$$

$$\sqrt{2} \cdot \lim_{x \rightarrow \infty} \frac{\frac{2}{x} - \frac{1}{x^2}}{5 + \frac{1}{x^2}} \leq \lim_{x \rightarrow \infty} f(x) \leq 2 \cdot \lim_{x \rightarrow \infty} \frac{\frac{2}{x} - \frac{1}{x^2}}{5 + \frac{1}{x^2}}$$

$$\sqrt{2} \cdot 0 \leq \lim_{x \rightarrow \infty} f(x) \leq 2 \cdot 0$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

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